Robertsson, J. O. A., Amundsen, L., and Sjøen Pedersen, Å.

Wavefield signal apparition, Part I: Theory

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Introduction

The Nyquist–Shannon sampling theorem describes how an analogue signal can be recovered from its discrete samples; for an infinite number of equidistant samples and given sample rate, $k_s$, perfect reconstruction is guaranteed provided that the underlying signal was bandlimited to $k \leq k_s/2$ (Shannon, 1949; Nyquist, 1928). The Nyquist–Shannon sampling theorem is equally relevant both to signals generated from a single source recorded on multiple receivers (receiver-side sampling) as well as signals generated from multiple sources and recorded at a single receiver (source-side sampling).

Assume that a wavefield $g$ is measured at a specific recording location for a source that is excited at different source positions along a straight line. The sampling theorem then dictates how the source locations must be sampled for a given frequency of the source and phase velocity of the wavefield.

Instead of using one source, we consider two (or more) sources that interfere during acquisition. The second source is triggered simultaneously or close in time with the first source while moving along another arbitrarily oriented straight line to excite the wavefield $h$. At the recording location the wavefields interfere and the sum of the two wavefields, $f = g + h$, is now measured. There is no known published exact solution to perfectly separate the wavefields $g$ and $h$ that were produced from each source from the combined measurement $f$.

Figure 1. Illustration of wavefield signal data gathers (in $\omega k$). (a, left): Conventional acquisition. (b, right): Acquisition following the principles of signal apparition.

Theory

The slowest observable (apparent) velocity of a signal along a line of recordings in any kind of wave experimentation is identical to the slowest propagation velocity in the medium where the recordings are made. As a result, after a spatial and temporal Fourier transform, large parts of the frequency-wavenumber ($\omega k$) spectrum inside the Nyquist frequency and wavenumber tend to be empty. In particular for marine reflection seismic data, the slowest observable velocity of arrivals corresponds to the propagation velocity in water (around 1500m/s). Figure 1a illustrates how all signal energy sits inside a “signal cone” (yellow) bounded by the propagation velocity of the recording medium.

In the wavefield experiment that we consider the source is excited sequentially for multiple source locations along a line while recording the reflected wavefield on at least one receiver. The source is characterized by its temporal signature. In the conventional way of acquiring data the source is excited using the same signature from source location to source location, denoted by integer $n$. Next, consider the alternative way of acquiring such a line of data using a periodic sequence of source signatures: every second source has constant signature and every other a signature which is a scaled (constant) or filtered function of the first source signature. Let this convolution filter be denoted by $a(t)$, with frequency-domain transform $A(\omega)$. Analyzed in the frequency domain, a receiver gather
(one receiver station measuring the response from a sequence of sources) recorded in this way, can be constructed from the following modulating function $m(n)$ applied to a conventional data set:

$$m(n) = \frac{1}{2}\left[1 + (1)^n\right] + \frac{1}{2} A\left[1 - (1)^n\right] = \frac{1}{2}\left[1 + e^{i\pi n}\right] + \frac{1}{2} A\left[1 - e^{i\pi n}\right].$$  \hspace{1cm} (1)

By applying the function $m$ in equation (1) as a modulating function to data $f(n)$ before taking a (normalized) discrete Fourier transform in space ($N$ uniformly spaced source points over $n$):

$$F(k) = \frac{1}{N} \sum_{n=0}^{N-1} f(n) e^{-i2\pi kn/N},$$

we obtain

$$H(k) = \frac{1}{N} \sum_{n=0}^{N-1} f(n)m(n) e^{-i2\pi kn/N} = \frac{1}{2}\left[1 + A\right]F(k) + \frac{1}{2}\left[1 - A\right]F(k-k_N),$$ \hspace{1cm} (2)

which follows from a standard Fourier transform result [wavenumber shift; Bracewell (1999)]. Identical results can also be derived using theory of sampling combs and considering two discrete sequences with twice the sampling distance but interleaved in space (Bracewell, 1999).

<table>
<thead>
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<th>$A(\omega)$</th>
<th>$H_-$</th>
<th>$H_+$</th>
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<tr>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
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<td>1/4</td>
<td>3/4</td>
</tr>
<tr>
<td>$e^{i\omega T}$</td>
<td>$(1 - e^{i\omega T})/2$</td>
<td>$(1 + e^{i\omega T})/2$</td>
</tr>
<tr>
<td>$1 + e^{i\omega T}$</td>
<td>$-e^{i\omega T}/2$</td>
<td>$1 + e^{i\omega T}/2$</td>
</tr>
</tbody>
</table>

Table 1. Mapping of signal to cone centred at $k=0$ ($H_-$) and cone centred at $k=k_N$ ($H_+$) for different choices of $A(\omega)$ for signal apparition in equation (2).

Equation (2) shows that the recorded data $f$ will be mapped into two places in the spectral domain as illustrated in Figure 1b [quantified in Table 1 for different choices of $A(\omega)$]. Part of the data will remain at the signal cone centered around $k=0$ (denoted by $H_-$ in the figure) and part of the data will be mapped to a signal cone centered around the Nyquist wavenumber $k_N$ (denoted by $H_+$). Note that by only knowing one of these parts of the data it is possible to predict the other using equation (2). Following Robertsson et al. (2015, A seismic shift: Wavefield signal apparition, paper submitted) we refer to this process as “wavefield apparition” or “signal apparition” in the meaning of “the act of becoming visible”. In the spectral domain, the wavefield caused by the periodic source sequence is nearly “ghostly apparent” and isolated (around $k_N$). We note that the word “spectrum” was introduced by Newton (1672) in relation to his studies of decomposition of light, when passed through a glass prism. This word is a variant of the Latin word “specter” which means “ghostly apparition”.

Signal apparition can be applied to all types of wavefield experimentation data. For instance data that are acquired for imaging of the Earth’s interior, such as acoustic, elastodynamic or electromagnetic data. A particular application of interest that can be solved using wavefield apparition is that of simultaneous source separation. Assume that a first source with constant signature is moved along a straight line with uniform sampling of the source locations where it generates the wavefield $g$. Along another straight line a second source is also moved with uniform sampling. Its signature is varied for every second shot according to the simple deterministic modulating sequence $m(n)$, generating the wavefield $h$. The summed, interfering data $f = g + h$ are recorded at a receiver location. In the frequency-wavenumber domain, where the recorded data are denoted by $F = G + H$,
the $H$-part is partitioned into two components $H_+$ and $H_-$ with $H = H_+ + H_-$. The $H_-$ component is “ghostly apparent” and isolated around the Nyquist-wavenumber (Figure 1b). Furthermore, $H_-$ is a known, scaled function of $H$. The scaling depends on the chosen $A(\omega)$ function [equation (2)], and can be deterministically removed, thereby producing the full appearance of the transformed wavefield $H$. When $H$ is found, then $G = F - H$. Inverse Fourier transforms yield the separate wavefields $g$ and $h$ in the time-space domain. The concept can be extended to the simultaneous acquisition of more than two source lines by choosing different modulation functions. Acquiring a source line where the first two source locations have the same signature, followed by two with signature modified by the function $A(\omega)$ and then repeating the pattern again until the full source line has been acquired, will generate a signal cone centered around $k_N/2$.

Figure 1b also illustrates a fundamental limitation of signal apparition. The $H_+$ and $H_-$ parts are separated within the blue and yellow lozenge-shaped regions in Figure 1b. In the green triangle-shaped parts they interfere and can no longer be separately predicted without further assumptions. In practice this means that the spatial sampling requirement becomes stricter compared to the case of an experiment with a single source. Note, however, that there is always a frequency below which the two signal cones separate perfectly. For the high-frequency band of the data we have therefore shifted the simultaneous source problem to another well-known problem of dealiasing and reconstruction of higher frequencies from lower frequencies where different techniques exist (Spitz, 1991).

Applications of seismic apparition

*Simultaneous source acquisition.* In a companion paper (Sjøen Pedersen, Å., Amundsen, L., and Robertsson, J. O. A., 2016, Wavefield signal apparition, Part II: Application to simultaneous sources and their separation: Expanded abstract submitted to the EAGE annual meeting) we discuss the application of seismic apparition to simultaneous source acquisition to increase productivity and/or obtain a better source-side sampling of the seismic data. The case where $A = -1$ (acquisition of data where the source signature flips polarity between source locations) may appear to be the optimal choice as it fully shifts all energy from $k=0$ to $k_N$ (see Table 1). However, for many applications (e.g., marine air gun sources) this is not feasible as it requires the ability to flip polarity of the source signal. Instead, we consider the much more practical case where every second source is excited a time shift $T$ later compared to neighbouring recordings: $A = e^{i\omega T}$ or $A = \delta(t-T)$. We note that although the case where $A = 0$ (source excited every second time only) results in somewhat reduced sub-surface illumination, it may be a straightforward way to acquire simultaneous source data.

*Residual shot noise attenuation.* By staggering shot times between flip and flop shots as described above, residual shot noise can be “apparated” and removed.

*Seismic interference cancellation.* If the dominant azimuth of seismic interference is known, it is possible to shift the acquired data so that it appears as far as possible from the seismic interference noise in $\omega k$ (e.g., noise centered at $k=0$ and signal at $k_N$). The application requires real-time knowledge of exact firing times of the interfering vessel.

*Modelling, imaging and inversion.* Wavefield apparition lends itself for straightforward applications in for instance finite-difference simulations where one of course has full control over source signatures. We note that through modeling it is also possible to tighten up the signal cone in Figure 1a and 1b enabling significant computational savings by means of using wavefield apparion.

*Source side deghosting* has been an unsolved problem since van Melle and Weatherburn (1953) dubbed the reflections from energy initially reflected above the level of the source, by optical analogy, ghosts. Figure 2 shows how wavefield apparition is applied to remove the source-side ghosts on a 2D
finite-difference synthetic data set acquired over a model resembling North Sea geology. Figure 2a shows the input common receiver gather data acquired for a node on the seafloor with source at 10m depth whereas Figure 2b shows the deghosted result which is close to perfect compared to our reference solution (Figure 2c). Such an excellent performance is obtained easily in cases where the blue and yellow lozenge-shaped regions in \( \omega_k \) space illustrated in Figure 1b do not interfere. In practice, a coarser source sampling may be required and we suggest the application of existing techniques to mitigate aliasing such as reconstruction with priors (e.g., Spitz, 1991).

![Figure 2](image)

**Figure 2** Source-side deghosting example. (a, left): Input data acquired with 10m source depth. (b, middle): Source-side deghosted data. (c, right): Reference solution.

**Conclusions**

We have presented a new theory for sampling of discrete signals referred to as signal apparition. The concept suggests alternative ways to acquire seismic data to solve a wide range of long-standing problems in seismic data acquisition and processing including simultaneous source acquisitions, seismic interference cancellation, residual shot noise attenuation, modelling and source deghosting. We illustrated the application of seismic apparition to remove source ghosts from data on a 2D numerical example. However, wavefield apparition makes no assumption about the structure of the medium of propagation and is directly generalized to complex 3D data acquired along single lines.

**Acknowledgements**

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**References**


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